Introduction to Probability

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Interpreting probability

- What is probability of:
  toss a fair coin = 1/2
  pass this class with an A
  cowboys win super bowl

- 0, 1

- The probability scale

- THE RELATIVE FREQUENCY VIEWPOINT
- THE SUBJECTIVE VIEWPOINT

THE RELATIVE FREQUENCY VIEWPOINT

Probability is approximately

#times event occur / # of experiments

Probability of an event in some process.

i.e. Process of rolling a dice, when sum of the two numbers is equal to 6.

Probability (sum is 6) is approximately

#6s / #tosses

THE SUBJECTIVE VIEWPOINT

- Used when the event can not be repeated
  - i.e. Probability of earning an A in this class

Measuring probabilities by means of a calibration experiment

Generally hard to measure

- Probability of events that are rare to occur
- Probability of events that are very likely to occur
- Probability of event that might or might not occur

Calibration experiment

- Chip-in-bowl experiment
  - Probability of red chip is drawn
    - 1 red and 9 white
    - 3 red and 7 white
    - 10 red
    - 10 white

Bidding game

- BET 1, white Christmas
- BET 2, draw a red in a bowl with 5 red and 5 white

- BET 1, white Christmas
- BET 2, draw a red in a bowl with 7 red and 3 white

- Therefore probability of white Christmas lies between .5 and .7

How can we convert odds to probabilities?

Probability (event) = B / B+A

Interpreting odds

the following is a paragraph from Roxy Roxborough's *Odds and Ends* column in *USA Today* .

EYE OF THE TIGER: Sensational PGA tour rookie Tiger Woods has been installed an early 9-5 favorite to win the most cash at the Skins Game at the Rancho La Quinta Golf Course in La Quinta, Calif., Nov. 30-Dec. 1. The charismatic Woods will have to contend with Fred Couples, the close 2-1 second choice, as well as the still dangerous Tom Watson, 3-1, and power stroker John Daly, 9-2.

How can we convert odds to probabilities?

Probability (event) = B / B+A

Therefore

5 / 5 + 9 = 5/14 = .3571

The sample space

Listing all possible outcomes:

- Sample space sum of two sides of die?

- Sample space, deck of cards?

Basic probability rules

Outcomes:	Max	Sue	Joe	Mary
Under 30 min	3	2	4	0
30 to 35 min	-1	3	4	2
35 to 40 min	4	4	1	8
Over 40 min	4	3	1	0

Basic probability rules

Driving to work example:

Outcome

1. Less than 30 minutes
2. 30 to 35 minutes
3. 35 to 40 minutes
4. Greater than 40 minutes

Basic probability rules

- Any probability assigned must be a nonnegative number.
- The probability of the sample space (the collection of all possible outcomes) is equal to 1.
- If you have two outcomes that can't happen at the same time, then the probability that either outcome occurs is the sum of the probabilities of the individual outcomes.

Equally likely outcomes

- 1/N

00 01 02 03 04 05 06 07 08 09
10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29

30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49
50 51 52 53 54 55 56 57 58 59
60 61 62 63 64 65 66 67 68 69
70 71 72 73 74 75 76 77 78 79
80 81 82 83 84 85 86 87 88 89

90 91 92 93 94 95 96 97 98 99

Constructing probability table by listing outcomes

- W1 W2 W3 M1 M2

COMMITTEE	PROBABILITY
W1, W2	1/10
W1, W3	1/10
W2, W3	1/10
M1, W1	1/10
M1, W2	1/10
M1, W3	1/10
M2, W1	1/10
M2, W2	1/10
M2, W3	1/10
M1, M2	1/10

Constructing a probability table by listing outcomes

COMMITTEE	# OF WOMEN	PROBABILITY
W1, W2	2	1/10
W1, W3	2	1/10
W2, W3	2	1/10
M1, W1	1	1/10
M1, W2	1	1/10
M1, W3	1	1/10
M2, W1	1	1/10
M2, W2	1	1/10
M2, W3	1	1/10
M1, M2	0	1/10

Constructing a probability table by listing outcomes

# OF WOMEN	PROBABILITY
0	1/10
1	6/10
2	3/10

Constructing a probability table by simulation

COIN TOSSES																				**NUMBER OF HEADS**
H	T	T	T	H	H	H	H	T	H	T	H	H	T	H	H	H	T	H	T	12

COIN TOSSES																				NUMBER OF HEADS
H	T	T	T	H	H	H	H	T	H	T	H	H	T	H	H	H	T	H	T	12
T	H	T	T	T	H	H	H	T	H	H	H	H	T	H	H	H	H	T	H	13
H	H	H	H	T	T	H	T	T	H	H	T	T	T	T	H	T	H	T	T	9
T	T	H	T	H	T	T	H	T	T	T	T	H	H	T	H	H	T	H	H	9
T	H	H	T	H	T	H	T	T	T	T	H	H	H	T	H	H	H	T	T	10
H	T	T	H	H	T	H	H	T	T	T	T	T	H	T	T	T	H	T	H	8
H	H	T	H	H	T	H	H	H	T	H	T	H	H	T	H	H	H	H	H	15
T	T	T	T	H	H	H	T	H	H	H	T	H	H	T	H	T	H	T	T	10
T	T	H	T	H	H	H	H	T	T	T	H	H	T	T	H	T	T	H	T	9
H	H	T	T	T	T	T	T	H	H	T	H	T	H	T	H	H	H	H	H	11

Demo by use a stem plot table

Probabilities of “or” and “not” events

- The addition rule
  - To compute or event probabilities
- The complement rule
  - To compute not event probabilities

Average value of probability distribution (mean value)

AMOUNT YOU WIN	PROBABILITY
20	12/38
-10	26/38

AMOUNT YOU WIN	PROBABILITY	PRODUCT
20	12/38	(20)(12/38) = 240/38
-10	26/38	(-10)(26/38) = -260/38
SUM		-20/38 = -.53

An average value of probability distribution

20 -10 -10 -10 20 -10 -10 20 -10 -10 -10 -10

20 20 20 20 20 -10 20 -10 20 -10 -10 -10

20 20 -10 -10 20 -10 -10 -10 -10 -10 -10 20

20 -10 -10 -10 -10 -10 20 -10 -10 20 20 20

-10 -10 20 -10 -10 -10 20 -10 -10 -10 -10 20

-10 20 -10 -10 20 -10 -10 -10 -10 -10 -10 -10

-10 -10 20 -10 -10 20 -10 -10 -10 -10 -10 20

-10 -10 20 -10 -10 20 -10 20 -10 -10 -10 20

-10 – 10 20 -10